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Mean-Field Limits of the Quantum Potts Model

A. Cant and Paul A. Pearce

Department of Theoretical Physics, Research School of Physical Sciences, The Australian National University, Canberra, A.C.T., 2601, Australia

Abstract. We consider the q-component quantum Potts model on a d-dimensional cubic lattice with symmetry breaking and transverse fields. The model is solved exactly in two special limiting cases: 1) the infinite lattice-dimensionality ($d \to \infty$) limit and 2) the limit of infinitely-weak, long-range interactions of Kac type. In each case the resulting free energy and its first partial derivatives (order parameters) are shown to be identical to the corresponding mean-field expressions.

1. Introduction

The Potts model is a model of central interest in statistical mechanics as is evidenced by the recent and extensive review article by Wu [1]. Although this model is a simple generalization of the 2-component Ising model to a q-component model, it exhibits much richer critical behaviour. Of particular interest is the order of the phase transition as one varies the lattice dimension d and the number of components q, regarded as continuous parameters. Mean-field theory [2] predicts a continuous transition for $q \leqq 2$ and a first-order transition for all $q > 2$, independent of the lattice dimension d. However, Baxter's exact result [3] in two dimensions shows a continuous transition for $q \leqq 4$ and a first-order transition for $q > 4$. In general, it is now believed that there exists a critical value $q_c(d)$, with a non-trivial dependence on the dimension d (see Fig. 2 in [1]), such that the mean-field prediction is correct for $q > q_c(d)$. In addition, renormalization-group arguments [4] indicate that the mean-field predictions are correct for $d > 4$. It is thus known that $q_c(2) = 4$ and $q_c(d) = 2$ for $d > 4$. An obvious question is what is the value of $q_c(d)$ for $d = 3$, in particular, is $q_c(3)$ greater than or less than 3? For some time the usual series expansions [5] and renormalization-group analyses [6] gave conflicting answers, but the weight of opinion now seems to be that $q_c(3) < 3$, that is, in three dimensions the 3-component Potts model undergoes a first-order transition.

Recently, a new attack has been made on these problems by looking at the quantum Hamiltonian (field theory) version [7] of the Potts model. Mean-field