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The Casimir Operators of Inhomogeneous Groups

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Abstract. We have found the number of invariant operators for the inhomogeneous groups IGL(n, R), ISL(n, R), ISO(p, q), IU(p, q), ISU(p, q), ISp$(2n)$, i.e. the inhomogeneous groups with the classical homogeneous subgroups, and also for the Weyl group W(p, q). For some special cases explicit forms of the invariant operators are obtained. We also discuss the methods applied, together with problems in some cases, possible further developments and relevance for the supersymmetric theories.

1. Introduction

One of the main characteristics of a group and the corresponding Lie algebra is the set of invariant operators, i.e. operators which have polynomial structure and are usually called the Casimir operators. Their construction is the first step in finding the representations of the group. In addition, algebraic invariants have immediate significance for constructing invariant equations, derivation of mass formulas and so on.

For the inhomogeneous groups the problem of searching for the Casimir operators and their eigenvalues has been solved in a whole series of works (see, e.g., [1, 2]). Less studied is the question of invariants for the inhomogeneous groups. The Casimir operators for the Poincaré group are well known and the Casimir operators for some other inhomogeneous groups have been found: ISL$(6, C)$ [3] and IU(n) with different subgroups of translation [4].

Inhomogeneous groups find important application in the gauge theory of gravitation. Kibble [5] generalized the gauge theory for the gravitational field using as the gauge group the (inhomogeneous) Poincaré group instead of the (homogeneous) Lorentz group which had been introduced by Utiyama [6]. This allowed, in contrast with Utiyama's theory, to introduce within the framework of only principle of local invariance a set of inhomogeneous quantities (vierbeins and relations among them) in a natural way. There exist analogous theories based on the inhomogeneous version of GL(n) and also the Weyl and the conformal groups.