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Monopoles and Spectral Curves for Arbitrary Lie Groups

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Abstract. The definition of the spectral curve of a monopole is extended to any connected, compact, simple Lie group K. It is found there are rank K curves whose degrees are related to the topological weights of the monopole.

1. Introduction

A lot is now known about static monopoles in Euclidean three-space for the gauge group SU(2) [5, 7, 11, 13, 14]. These are solutions of the Bogomolny equations, in the Bogomolny-Prasad-Sommerfield (BPS) limit and are classified by an integer k, the topological weight. For $k=1$ the only monopoles are all translates of the spherically symmetric Prasad-Sommerfield monopole [10]. For higher k axially symmetric solutions have been constructed [14, 12, 5] and recently shown to be regular [8]. The existence of a large number of solutions for each k with no special symmetry has been shown by Taubes [13] and Weinberg has shown that they depend on $4k-1$ parameters [15].

By using the "twistor methods" of Penrose, Ward and Atiyah, Hitchin [7] has shown that each monopole has associated to it a real, algebraic curve of degree $2k$, the spectral curve, from which the monopole can be reconstructed.

For groups K larger than SU(2) the monopoles are classified by r topological weights $(m_1, \ldots, m_r)$, where r is the rank of K. Taubes has shown [13] that there exist monopoles for any K and $(m_1, \ldots, m_r)$ when all the m_i are non-negative, and Weinberg has calculated the number of parameters that these solutions depend upon [15].

The same twistor methods can be applied to these monopoles, and we show that there exist r spectral curves $S_1, \ldots, S_r$ naturally associated to the vertices of the Dynkin diagram. Again the curves have degree $2m_i$. If the intersections $S_i \cap S_j$ are finite whenever i and j are joined on the Dynkin diagram (if the curves are reducible $S_i \cap S_j$ may contain a component), then the monopole can be reconstructed from the spectral curves.