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Log Hölder Continuity of the Integrated Density of States for Stochastic Jacobi Matrices

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Abstract. We consider the integrated density of states, $k(E)$, of a general operator on $\ell_2(\mathbb{Z}^\nu)$ of the form $h = h_0 + v$, where $(h_0u)(n) = \sum_{|i|=1} u(n+i)$ and $(vu)(n) = v(n)u(n)$, where v is a general bounded ergodic stationary process on $\mathbb{Z}^\nu$. We show that $|k(E) - k(E')| \leqq C[-\log(|E-E'|]^{-1}$ when $|E-E'| \leqq \frac{1}{2}$, The key is a "Thouless formula for the strip."

1. Introduction

In this paper, we discuss general multidimensional stochastic Jacobi matrices. Explicitly, let (Ω, μ, Σ) be a probability measure space on which $\mathbb{Z}^\nu$ acts, that is, ν commuting measure preserving invertible transformations, $T_1,\dots,T_\nu$ are given. If $n = (n_1,\dots,n_\nu) \in \mathbb{Z}^\nu$, we let $T^n = T_1^{n_1} \dots T_1^{n_\nu}$. We suppose that the action is ergodic. Fix a measurable real valued function f on Ω and let $v_\omega(n) \equiv f(T_\omega^n)$. On $\ell_2(\mathbb{Z}^\nu)$ let h_0 be the finite difference Laplacian given by

$$(h_0u)(n) = \sum_{|\delta|=1} u(n+\delta), \tag{1.1}$$

where the sum is over the 2ν nearest neighbors of n. Let v_ω be the diagonal operator $(v_\omega u)(n) = v_\omega(n)u_\omega(n)$. We consider the operators

$$h_\omega = h_0 + v_\omega. \tag{1.2}$$

In the bulk of this paper, we assume that the function $f(\omega)$ is bounded. In fact our main theorem extends, with minor modifications of the proof, to the case that $\ln(|f|+1)$ is in L^1; these modifications are sketched in Sect. 3.

Examples of interest include the following cases: (a) The periodic case where Ω is finite and each T_i is periodic. (b) The almost periodic case where Ω is a compact metric space and the T's are isometries (see e.g. [3]). (c) The random case where the process $v_\omega(n)$ is a set of independent, identically distributed random variables.

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