Commun. Math. Phys. 90, 79–100 (1983)

Communications in
Mathematical
Physics

Geometry of Multidimensional Universes

R. Coquereaux* and A. Jadczyk**,***

CERN, CH-1211 Geneva 23, Switzerland

Abstract. Let G be a compact group of transformation (global symmetry group) of a manifold E (multidimensional universe) with all orbits of the same type (one stratum). We study G invariant metrics on E and show that there is one-to-one correspondence between those metrics and triples $(g_{\mu\nu}, A_{\mu}^{\hat{a}}, h_{\alpha\beta})$, where $g_{\mu\nu}$ is a (pseudo-) Riemannian metric on the space of orbits (space-time), $A_{\mu}^{\hat{a}}$ is a Yang-Mills field for the gauge group $N|H$, where N is the normalizer of the isotropy group H in G, and $h_{\alpha\beta}$ are certain scalar fields characterizing geometry of the orbits (internal spaces). The scalar curvature of E is expressed in terms of the component fields on M. Examples and model building recipes are also given. The results generalize those of non-abelian Kaluza-Klein theories to the case where internal spaces are not necessarily group manifolds.

1. Introduction

First special, and later general, theories of relativity invoked a picture of the universe as being modelled on a four-dimensional space-time manifold. On the other hand, in order to describe regularities of discrete quantum numbers characterizing elementary particles, a concept of "internal" (as opposed to "external", i.e., space-time) symmetry, and with it that of internal space, was introduced. The idea behind what we call a "multidimensional universe" (denote it by E) is that external and internal spaces are nothing but two aspects of one geometrical entity E, and all elementary forces in nature should be but a reflection of a unique geometry. By some, not yet fully understood, mechanism, certain configurations of a simple, multidimensional field theory are distinguished, and give rise to a "spontaneous compactification" of extra dimensions (see [1] and references therein). This idea is at the root of the so-called "dimensional

* On leave of absence from Centre de Physique Théorique, F-13288 Luminy, Marseille, France

** On leave of absence from Institute of Theoretical Physics, University of Wroclaw, Cybulskiego 36, PL-50-205 Wroclaw, Poland

*** Work done in the framework of the Project MRI. 7. of the Polish Ministry of Science, Higher Education and Technology