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H-Surfaces in Lorentzian Manifolds*

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Abstract. We consider surfaces of prescribed mean curvature in a Lorentzian manifold and show the existence of a foliation by surfaces of constant mean curvature.

0. Introduction

Surfaces of prescribed mean curvature, that is what we mean by H-surfaces, are of great physical importance both in the case of a proper Riemannian manifold as well as in a Lorentzian manifold. While H-surfaces in proper Riemannian manifolds, especially in the Euclidean space $\mathbb{R}^n$, have been studied extensively, little is known in the Lorentzian case, except when the manifold is the Minkowski space. Then, there are the papers of Calabi [CA] and Cheng and Yau [CY] on the Bernstein theorem for entire maximal surfaces, the result of Treibergs [TA] on entire surfaces of constant mean curvature, and the paper of Bartnik and Simon [BS] on the Dirichlet problem for surfaces with bounded mean curvature.

For non-flat Lorentz manifolds only local existence results via perturbation arguments, or results concerning the uniqueness are known, cf. [BF1, 2; CB; CFM; GO; MT].

In this paper we consider a connected, oriented, and time-oriented, globally hyperbolic Lorentz manifold M of dimension $(n+1)$.

In the first part of this paper, Sects. 1–5, we consider the Dirichlet problem for bounded H-surfaces. Assuming in this case that M is topologically a product,

$$M = N \times I, \tag{0.1}$$

where I is an interval and N an n-dimensional complete Riemannian manifold, such that the metric in M is given as

$$ds^2 = \psi(-dt^2 + g_{ij}(x)dx^i dx^j) \tag{0.2}$$

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