Statistics of Yang-Mills Solitons

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Abstract. We determine the statistics and the spin of isolated Yang-Mills monopoles in eigenstates of their electric and magnetic charge. Exchange of solitons is defined using the translation operator of a companion paper; and under exchange, state vectors representing N identical solitons change sign precisely when the angular momentum of each soliton is half-integral.

I. Introduction

The symmetries of classical mechanics can be understood as expressing a freedom in the relation between a subsystem and its environment. When this environment is neutral in some neighborhood of the subsystem – when, for example, the forces due to external matter are small – it makes sense to translate or rotate the subsystem without altering its internal state. Then the invariance of the total system's action under such changes implies the conservation of the momentum and angular-momentum of the subsystem.

   In gauge theories, an invariance of the vacuum (neutral environment) under some global gauge-transformation implies a further freedom, one which is exercised by applying the transformation to the subsystem *but not* to the environment, just as one rotates a subsystem by applying "rotation" to it but not to the objects around it. Here again symmetry leads to conservation laws, for example to the conservation of the electric charge of the subsystem in theories with an unbroken phase-invariance.

   When such "internal" gauge-symmetries are present alongside spacetime ones any symmetry operation can be composed with an "internal" one without affecting its geometrical content. In particular the operation of exchanging a pair of indistinguishable subsystems (which might be single particles) via a sequence of translations through the vacuum is ambiguous until one specifies which of the gauge-equivalent versions of translation is to be used at each stage.