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Kinematics of Yang-Mills Solitons

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Abstract. In SU(2) Yang-Mills theory, the N-monopole configuration space is a bundle with fibers isomorphic to $U(1) \times \ldots \times U(1)$, and state vectors for which each monopole has charge $n\frac{e}{2}$ are homogeneous of degree n with respect to each U(1). Translations and rotations are defined for individual monopoles in the N-monopole space. The commutator of two translations is found to be a U(1) transformation that agrees for large monopole separation with the analogous phase change accompanying the translation of a charged point particle in an external magnetic field. The theory developed here is applied in a companion paper to prove a spin-statistics theorem for monopoles in SU(2) Yang-Mills theory.

1. Introduction

We examine here the kinematics of non-overlapping monopoles in an SU(2) Yang-Mills-Higgs theory [1]. In Sect. II we briefly review the quantum theory of Yang-Mills monopoles in the Schrödinger representation [2, 3]. We observe first that for a single monopole, the configuration space $\mathscr{C}$ on which the state functional ψ takes its values is a principle U(1) bundle; and state vectors of charge $Q_E = n\frac{e}{2}$ are homogeneous functionals of degree n on the bundle, where $\frac{e}{2}$ is the smallest unit of electric charge. We define rotations and translations in the one-monopole sector by requiring that these transformations leave unchaged the asymptotic behaviour of the fields. It then follows that the state space includes states for which the corresponding angular momentum operators have half integral values.

In analyzing monopole kinematics, our principal aim is to define translations and rotations of individual monopoles in the presence of other monopoles, and

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