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The Scattering of Certain Yang–Mills Fields*

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Abstract. The Yang–Mills fields considered by us in an earlier paper are asymptotically non-interacting. Also any free field is an incoming field for some Yang–Mills field.

Introduction

Interest in the classical solutions of the Yang–Mills equations in Minkowski space has grown in recent years. The definitive global existence theorem (solution of the Cauchy problem) has been found by Eardley and Moncrief [3], following the local theorem of Segal [8]. Other existence results and methods appear in [6], [4] and [1]. However, the scattering problem has so far been left untouched. Christodoulou's transform method can be used to derive some decay properties. In [5] we showed how the conformal invariance directly implies certain asymptotic properties of the fields, in particular, local decay of the energy.

In Sect. 1 of this paper we use these asymptotic properties, and the special properties of the class of solutions discussed in [6], to prove that these solutions α are asymptotically free fields in the energy norm. In Sect. 3 we show that any free field α_- of our special type is the incoming field of some α. In Sect. 2 we derive some explicit pointwise bounds needed in the proof.

Our class of solutions is defined by a condition of the Polyakov-t'Hooft type for the gauge group SU(2). We emphasize that there is no restriction on the size of the solutions we consider. Specifically, the gauge potentials have the form

$$A^k = \alpha(r,t)v^k \quad (k=1,2,3), \quad A^0 = 0.$$

where t is time, $x \in \mathbb{R}^3$, $r = |x|$ and v^k are certain vectors (see [6]). Such a field belongs to both the temporal and Coulomb gauges. The field equations reduce to a single scalar wave equation (see (1) below). Although this equation appears rather innocuous, its asymptotic analysis is surprisingly non-trivial due to the singularity at the origin. The free fields we consider are simply solutions of the Yang–Mills

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