Commun. Math. Phys. 89, 445–464 (1983)

Communications in Mathematical Physics

Initial Boundary Value Problems for the Equations of Motion of Compressible Viscous and Heat-Conductive Fluids

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Abstract. The equations of motion of compressible viscous and heat-conductive fluids are investigated for initial boundary value problems on the half space and on the exterior domain of any bounded region. The global solution in time is proved to exist uniquely and approach the stationary state as $t \to \infty$, provided the prescribed initial data and the external force are sufficiently small.

1. Introduction

The motion of viscous compressible fluids is described by the system of five equations for the density ρ, the velocity $u = (u^1, u^2, u^3)$ and the temperature θ:

$$\rho_t + (\rho u^j)_{x_j} = 0,$$

$$u_t^i + u^j u_{x_j}^i + \frac{1}{\rho} p_{x_i} = \frac{1}{\rho} \{\mu(u_{x_j}^i + u_{x_i}^j) + \mu' u_{x_k}^k \delta^{ij}\}_{x_j} + f^i, \quad i = 1, 2, 3,$$

$$\theta_t + u^j \theta_{x_j} + \frac{\theta p_\theta}{\rho c_V} u_{x_j}^j = \frac{1}{\rho c_V} \{(\kappa \theta_{x_j})_{x_j} + \Psi\}, \tag{1.1}$$

where p is the pressure, μ is the viscosity coefficient, μ' is the second coefficient of viscosity, κ is the coefficient of heat conduction, c_V is the specific heat at constant volume, all of which are known functions of ρ and θ, and Ψ is the dissipation function:

$$\Psi = \frac{\mu}{2}(u_{x_k}^j + u_{x_j}^k)^2 + \mu'(u_{x_k}^k)^2.$$

We consider the initial boundary value problem of (1.1) in the region $t \geqq 0, x \in \Omega$. The boundary condition is supposed

$$u|_{\partial\Omega} = u|_\infty = 0, \quad \theta|_{\partial\Omega} = \theta|_\infty = \bar{\theta}, \quad t > 0, \tag{1.2}$$

* Both authors are partially supported by the University of Wisconsin–Madison, Mathematics Research Center