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On the Energy Spectrum and Parameter Spaces of Classical CP^n Models*

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Abstract. It is known that a large class of smooth solutions of CP^n models can be constructed starting from holomorphic maps of an algebraic curve into complex projective spaces. Here we apply results from algebraic geometry to describe the energy spectrum and the parameter spaces for such models.

1. Introduction

Harmonic maps theory has been recently applied to a well known problem of mathematical physics, namely the CP^n models. These have been quite extensively studied in the physical literature not because of direct physical applications, which seem limited in number and perspective, but in that they exhibit interesting phenomena common to Yang-Mills theories both at the classical and at the quantum level [1, 2].

One of these coincidences has to do with the semi-classical domain of the theories, whereby the properties of classical solutions of the elliptic form of the field equations are examined. From the physical point of view, the interest in such "pseudoparticle" solutions of classical field equations was first pointed out by Polyakov in connection with the infrared problem in the quantum theory of Yang-Mills fields. The SU(2)-instanton solutions were first found by Belavin et al. [3] for Yang-Mills equations. Shortly afterwards, the same type of reasoning was applied by Belavin and Polyakov [4] to the standard SO(3)-invariant σ-model. This was subsequently generalized by Eichenherr [5] to $SU(n+1)$-invariant σ-models, which were christened CP^n-models by D'Adda et al. [2].

From the mathematical point of view, it was immediately clear that both these generalizations, as well as the original σ-model, could be considered as particular examples of harmonic problems. An extensive mathematical literature exists on this topic [6] from which we shall recall below some definitions and results. One of the basic outcomes of these developments is that classical solutions

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