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An Adiabatic Theorem Applicable to the Stark Effect

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Abstract. We prove an adiabatic theorem applicable to an atom evolving in a slowly varying electric field. This yields an operational estimate of the tunneling rate even for systems where complex scaling techniques are not applicable.

1. Introduction

The discovery of the spectral theorem by Stone and von Neumann in 1930 provided an extremely powerful tool for early quantum mechanics, and enabled a deep investigation both of bound states and of scattering theory to be initiated. At the computational level these have been well understood for many years, and their theoretical analysis is now largely complete. On the other hand the study of metastable states (resonances) has made much slower progress, and it is only with the introduction in the last decade of ideas known collectively as complex scaling ([1, 2, 4, 5, 12, 13] and references there in) that real insight has been achieved.

A standard application of complex scaling is to the Stark effect [4, 5, 13] for the hydrogen atom, with Hamiltonian

$$H_E = -\Delta - \frac{2}{|x|} + Ex_3.$$

Although this operator is known to have no bound states for $E \neq 0$, complex scaling leads to the discovery of a complex eigenvalue λ_E whose associated eigenfunction ϕ_E is not square-integrable. The real part of λ_E is often identified as the perturbed energy level of the hydrogen atom, while the imaginary part, which is exponentially small, is identified as the decay or tunneling rate.

One way of trying to justify these identifications operationally would be to consider the evolution of the ground state of the hydrogen atom if the external field is switched on adiabatically. Thus one examines the form of the solution of $\psi'(t) = -iH_{E(t)}\psi(t)$ if $E(0)=0$, and $E(t)$ varies very slowly. The adiabatic theorem of Kato [6] is not applicable here because H_E has no bound states for $E \neq 0$. The obvious guess that $\psi(t)$ should be near to the resonance eigenfunction $\phi_{E(t)}$ up to a phase is clearly also impossible if one measures nearness in L^2 norm. We do not attempt to