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Exterior Complex Scaling and the AC-Stark Effect in a Coulomb Field

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Abstract. By means of the exterior complex scaling of B. Simon an existence proof of resonances is given for the time-dependent Schrödinger equation

$$i\frac{\partial\psi}{\partial t}=(-\Delta+V+\mu x_1\cos\omega t)\psi,$$

where V belongs to a class of potentials which includes the Coulomb one. The resonance width is given by the Fermi Golden Rule to second order perturbation theory and is nonzero for μ small and almost every ω.

1. Introduction

The time dependent Schrödinger equation

$$i\frac{\partial\psi}{\partial t}=(-\Delta+V(x)+\tfrac{1}{2}\mu x_1\cos\omega t)\psi \tag{1.1}$$

describes the so-called AC-Stark effect, i.e. the quantum motion of a particle of mass $\frac{1}{2}$, unit charge, coordinates $x=(x_1,x_2,x_3)\in\mathbb{R}^3$ in a potential field $V(x)$ under the action of an oscillating electric field of strength $\frac{1}{2}\mu\cos\omega t$, $\mu\in\mathbb{R}$, directed along the x_1 axis. Here $\omega\in\mathbb{R}_+$, and $\hbar=1$. If V is not only dilation analytic, but also satisfies some extra smoothness assumptions, Yajima [12] has obtained, through a synthesis of Floquet theory and complex scaling, a mathematical justification of the well-known physical picture associated with the time dependent perturbation problem (1.1) (see e.g. [5, 11]).

Let us describe in some detail the results of [12], because the purpose of this paper is to extend the main one, i.e. the existence of resonances, to a class of potentials sufficiently general to include the Coulomb one. Setting, as in [3]:

$$\begin{aligned}\psi(t,x)&=T(t)v(t,x),\\(T(t)f)(x)&=e^{-i(\omega^{-1}\mu x_1\sin\omega t-16\mu^{-2}\omega^{-3}\sin 2\omega t+8\mu^2\omega^{-2}t)}f(x_1-\mu\omega^{-2}\cos\omega t,x_2x_3),\end{aligned}\tag{1.2}$$

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