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Hyperbolicity of the 3+1 System of Einstein Equations

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Abstract. By a suitable choice of the lapse, which in a natural way is connected to the space metric, we obtain a hyperbolic system from the 3+1 system of Einstein equations with zero shift; this is accomplished by combining the evolution equations with the constraints.

Introduction

The success of the temporal gauge in Yang Mills theory for proving existence theorems (cf. [17, 18, 7]) has led us to look for an analogous gauge in general relativity. It is well known that neither normal gaussian coordinates, nor other arbitrary choices of lapse and shift lead to a hyperbolic evolution system for the conjugate unknowns g_{ij} and P^{ij}. However this system is very useful in numerical computations of space time models [16] as well as in certain quantization procedures ([2, 1]).

We show here that if we call "temporal gauge" a choice of time lines orthogonal to the space sections (choice always possible in a globally hyperbolic manifold) the conjugate unknowns satisfy a hyperbolic evolution system if we choose the lapse appropriately. The local existence and global uniqueness theorems[1], under their most refined form (cf. [6, 4]) can then be deduced with the help of the usual machinery constraints – Bianchi identities. The explicit geometric expression of the evolution system gives a better light to study global problems.

Let us remark, for numerical relativists, that the choice of lapse we propose with zero shift, has the merit to make stable the solution of the Cauchy problem, a property which could be useful in their constructions of dynamical models.

1. The 3+1 Equations

We recall the Einstein equations in the 3+1 form (see [15, 8, 1]), or the review article [9]). Let $S \times \mathbb{R}$ denote the space-time manifold; we choose the time-lines

1 A different approach which also does not use harmonic coordinates is due to DeTurk (preprint) that introduces an assigned 2-tensor