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Nonexistence of Quantum Fields Associated with Two-Dimensional Spacelike Planes

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Abstract. It is shown that in a relativistic quantum field theory satisfying Wightman's axioms, there are no nontrivial field-like operators, or even bilinear forms, associated to a two (or less)-dimensional spacelike plane in Minkowski space. This generalizes Wightman's result that fields can not be defined as operators at a point and stands in contrast to Borchers' result that field operators can be associated with one-dimensional timelike planes.

1. In this note we use analyticity properties associated with the Lorentz boosts in wedgelike spacetime regions (arising from Bisognano and Wichmann's identification of the modular automorphism group of wedge algebras with such boosts [1,2]) to prove that the polynomial operator algebras generated by the fields associated with a wedge and its causal complement are "irreducible" in a quantum field theory satisfying the Wightman axioms [3], so that, in a strong sense to be specified below, no nontrivial (unbounded) operator (or even bilinear form) on the physical Hilbert space can be associated with a two (or less)-dimensional spacelike plane. This latter point is a generalization of a result of Wightman [4] on the nonexistence of nontrivial field operators at a point, and is to be seen in contradistinction to a result of Borchers [5], that field operators can be associated with one-dimensional timelike planes. There are, of course, examples of fields at sharp times, i.e. associated with three-dimensional spacelike planes, e.g. the free scalar field.

2. To begin, we establish the framework and notation necessary for the statement and proof of the results. We assume the usual Wightman axioms, including Poincaré covariance, locality, spectrum condition and the uniqueness of the vacuum Ω (see [3] for further details), and we admit Bose and Fermi statistics. Let W_R and W_L denote the right and left wedges defined by

$$W_R = \{x \in \mathbb{R}^4 \,|\, x^1 > |x^0|\}, \quad W_L = \{x \in \mathbb{R}^4 \,|\, x^1 < -|x^0|\},$$

where x^0 is the time coordinate. Then $\mathscr{P}(W_R)$, respectively $\mathscr{P}(W_L)$, will signify the $*$-algebra consisting of all polynomials of field operators tested with functions from $\mathscr{S}(\mathbb{R}^4)$ that have support in W_R, respectively W_L. These operators are defined on