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A Rigorous Upper Bound in Electrostatics on a Random Lattice Ensemble*

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Abstract. We consider a Kirchhoff network on a random two-dimensional lattice with links and weights as previously specified, and a circular boundary of radius R. We show rigorously that the resistance between the central point and the boundary, averaged over all placements of the remaining sites with site density ϱ, is bounded above by

$$(4\pi)^{-1}[\ln(4\pi\varrho R^2)+1]+16[\tan^{-1}5^{-1/4}+5^{1/4}/(\sqrt{5}+1)^2]$$
$$\simeq(4\pi)^{-1}\ln(4\pi\varrho R^2)+12.0\,.$$

I. Introduction

In a series of recent papers [1–3], a formulation of field theory on a random lattice has been developed. The possibility was raised that this approach might lead to a true theory of nature and not merely a method of making numerical approximations to continuous field theory. According to this view, physical quantities might be obtained as averages over an ensemble of random lattices.

Such an idea invites a program of research in which almost every branch of mathematical physics may be examined to see how it is altered when reformulated along the lines of [1–3]. We have addressed ourselves here to electrostatics in two dimensions, and specifically to the finiteness of the self-potential of a charge.

When electrostatics in a vacuum is reformulated as a lattice theory, it becomes the problem of a network of capacitors joined at nodes which may bear charge. If a unit charge is placed at the origin, the potential at large distances will behave as in the continuous theory, but at short distances it will behave quite differently. In the continuous theory, the potential is singular at the origin; in the network, it remains finite even at the node on which the charge is placed. (In two dimensions, we make the potential vanish at a finite boundary so as to eliminate the logarithmic infinity at large distances.)

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