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Steepest Descent Path for the Microcanonical Ensemble-Resolution of an Ambiguity

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Abstract. The microcanonical entropy plays an essential role in the equilibrium statistical mechanics of gravitating systems. A peculiar feature of many of these systems is the existence of stable thermodynamic equilibrium configurations with negative heat capacities. Different methods have been developed for calculating the microcanonical entropy involving multivariate integrals of constraints and functional integrations. An apparent ambiguity between an approach due to Hawking and Gibbons, based on an entropy definition involving an inverse Laplace transform of the partition function, which they developed to treat quantum systems with gravity, and a different approach developed by Horwitz and Katz defining the entropy as an equal weight sum over a constant energy surface developed originally to treat Newtonian and classical GR systems is shown here to be spurious, at least at the level of quadratic fluctuations of all variables about the extremal solutions. The two approaches involve distinct contours for different orders of integration, each of which is shown to be the appropriate steepest descent path corresponding to the given order of investigation. Up to quadratic fluctuations both methods yield identical results. However, they represent different perturbation expansions for the gravitational modes of freedom with different radii of convergence. The discussion is made in terms of a particular convenient model, a system of point particles interacting via Newtonian forces, confined to a sphere, but results are quite general.

I. Introduction

Calculations using a microcanonical ensemble (MCE) involve many practical difficulties for virtually any system. On the basis of the equivalence of different ensemble which is valid for the usual uniform system, one commonly carries out calculations in the most convenient ensemble and then one obtains the entropy by means of a Legendre transformation. Thus, for classical systems, standard calculations use either the canonical ensemble (CE) or the grand canonical