

The Energy-Momentum Tensor near a Black Hole^{*}

M. S. Fawcett

Department of Mathematics, University of Otago, Dunedin, New Zealand

Abstract. The energy-momentum tensor for a conformally invariant scalar field near a Schwarzschild black hole in thermal equilibrium with radiation is found by a combination of analytical and numerical techniques. Calculations are performed in the Euclidean section of the spacetime, and divergences isolated using the heat kernel expansion. It is found that the results agree well with those of Candelas [1], but that there are significant differences from the Gaussian approximation of Page [2].

1. Introduction

This paper describes the calculation of the expectation value of the energy-momentum tensor, to one loop, for a conformally coupled scalar field near a Schwarzschild black hole in the Hartle-Hawking “vacuum” state – i.e. for a black hole in (unstable) equilibrium with a bath of thermal radiation. Much preliminary work has been done on this problem, and there is a large literature on black hole radiance and the quantum properties of black hole spacetimes, and on quantum field theory in more general curved spacetimes. The original work on black hole radiance was done by Hawking [3] and Hartle and Hawking [4]. Boulware [5], Unruh [6], and Wald [7] further discuss black hole emission and different possible vacuum states. Davies et al. [8], Christensen and Fulling [9], and Candelas [1] discuss the energy-momentum tensor of a scalar field near a black hole. Gibbons and Perry [10] discuss the thermal properties of black holes, and thermal Green functions. There are of course many other papers, references to which can be found in the reviews, e.g. by Gibbons [11, 12], De Witt [13, 14], and the recent book by Birrell and Davies [15]. These reviews also discuss methods of regularising the expressions for physical quantities in a general spacetime.

The calculation of $\langle T_{\mu\nu} \rangle$ could have been done years ago – the delay has probably been due to the (perhaps very reasonable) reluctance of people to get

* This paper was written while the author was at the DAMTP, Cambridge University, Silver Street, Cambridge, England