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Two-Dimensional Exactly and Completely Integrable Dynamical Systems

Monopoles, Instantons, Dual Models, Relativistic Strings, Lund-Regge Model, Generalized Toda Lattice, etc.

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Abstract. An investigation of two-dimensional exactly and completely integrable dynamical systems associated with the local part of an arbitrary Lie algebra $\mathfrak{g}$ whose grading is consistent with an arbitrary integral embedding of $3d$-subalgebra in $\mathfrak{g}$ has been carried out. We have constructed in an explicit form the corresponding systems of nonlinear partial differential equations of the second order and obtained their general solutions in the sense of a Goursat problem. A method for the construction of a wide class of infinite-dimensional Lie algebras of finite growth has been proposed.

1. Introduction

In papers [1] (see also [2, 3]) we proposed a general scheme for the construction of exactly and completely integrable dynamical systems in two-dimensional space associated with an arbitrary graded Lie algebra or superalgebra $\mathfrak{g}=\sum_{-\infty}^{+\infty}\oplus\mathfrak{g}_a$, and developed a group method to find general solutions to these systems. The method enables us to obtain closed expressions for the solutions. However, due to the absence of a general procedure for the description (finding the structure constants) of Lie algebras of "arbitrary position", it is not always possible to write the equations of the corresponding systems in an explicit form. Furthermore the formulation of the equations is essentially dependent on a choice of gauge constraints.

In view of the aforementioned reasons, the proposed algebraic construction was fully realized [4] [in the sense of explicit formulae for the general solutions and the equations themselves) for the dynamical systems with an abelian invariance subalgebra $\mathfrak{g}_0=\sum_{1}^{r}\oplus u(1)$, $r\equiv\operatorname{rank}\mathfrak{g}$, which have no ambiguity related with a choice of gauge constraint. Later on the basis of study of the Lie-Bäcklund group transformation general criteria of exact or complete integrability for the