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The Unitary Irreducible Representations of $\overline{\mathrm{SO}}_0(4,2)$

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Abstract. An exhaustive classification of all irreducible Harish-Chandra $\mathfrak{so}(4,2)$-modules, integrable to unitarizable projective representations of the conformal group, is established by infinitesimal methods: the classification is based

1) on the reduction upon the maximal compact subalgebra, associated with a lattice of points in $\mathbb{R}^3$, and

2) on a set of additional parameters upon which the eigenvalues of central elements of the enveloping algebra depend polynomially.

0. Introduction

The conformal group of Minkowski space has recurrently been a quite important tool in mathematical physics. Its unitary irreducible representations, true or projective ones, have been a favorite subject of research of many scientists [1]; but, in spite of the various methods used, there is no exhaustive list of them, at least to the author's knowledge [8].

The object of this paper is to give an exhaustive classification of the unitary dual of the universal covering of the connected component of the conformal group, $\bar{G}=\overline{\mathrm{SO}}_0(4,2)$; more precisely the problem studied is the (equivalent) Lie algebraic transcription of that statement: determine, up to infinitesimal equivalence, all Schur-irreducible representations π of $\mathfrak{g}=\mathfrak{so}(4,2)$, on a pre-Hilbert space of $\mathfrak{k}$-finite vectors ($\mathfrak{k}=\mathfrak{so}(4)\oplus\mathfrak{so}(2)$), such that $i\pi(X)$ is essentially self-adjoint for every X in $\mathfrak{g}$.

In this paper we shall only sketch the proof for the following reason: after solving the above problem for $\mathfrak{so}(3,2)$ [2], we developed a formalism for solving the $\mathfrak{so}(p,2)$ case, establishing necessary and sufficient conditions for unitarity [3]. Whereas this formalism would be too long to expose here in full detail, the final explicit formulas can be given rather concisely in the physically interesting case $p=4$.

The principle of the method used is the following: Let $\mathfrak{g}=\mathfrak{k}\oplus\mathfrak{p}$ be a Cartan decomposition of a real semisimple Lie-algebra, and consider a $\mathfrak{g}$-module $\mathscr{E}$ having