

## On the Interpretation of $1/f$ Noise

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**Abstract.** We propose a model of  $1/f$  noise based on a random walk in a random potential. Numerical support for the model is given, and physical applicability discussed.

### 1. Introduction

The frequency analysis of fluctuations in a number of physical phenomena exhibits a remarkable feature. It is found that the experimental power spectrum (i.e., essentially, the frequency distribution of the squared amplitude) behaves like  $f^{-1}$  at low frequency  $f$ . (This explains the name of  $1/f$  noise.) The prime example of this type of behaviour is seen in voltage fluctuations across a conductor carrying electric current (see Hooge et al. [1], Dutta and Horn [2] for reviews). One also observes  $1/f$  noise (sometimes called *flicker* noise) in such diverse questions as fluctuations of marine currents, or the temporal distribution of loudness in a musical recording (see for instance Press [3]). The  $1/f$  law sometimes extends over many decades of frequency, implying the existence of correlations over surprisingly long times for the systems considered. (Since the integrated spectrum would diverge logarithmically, one expects that there is a low frequency cutoff.)

It is easy to obtain a power spectrum  $\sim$  constant (independently distributed events) or  $\sim 1/f^2$  (independently distributed increments). The  $1/f$  law is more difficult to explain, especially that its universality requires an interpretation of general applicability. There is no natural time scale associated with pure  $1/f$  noise. Considerations of self-similarity therefore come naturally to mind (see Mandelbrot [4]) but something more specific is needed. Explanations based on spatial diffusion depend on special geometric assumptions (see Voss and Clarke [5] and Omnes [6])<sup>1</sup>. A guide towards understanding flicker noise in conductors is provided by the experimental fact that they are due to *equilibrium fluctuations* of the resistance (see [5, 1], and more detailed results about so-called  $\alpha$  noise in [1]).

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<sup>1</sup> Explanations using the deterministic noise associated with low-dimensional strange attractors are also in doubt (see Arrechi and Lisi [7], Beasley et al. [8]). See however remark (b) below