Commun. Math. Phys. 88, 581–600 (1983)

Communications in Mathematical Physics

On the Massive Sine-Gordon Equation in the Higher Regions of Collapse

Francesco Nicolò

Istituto di Matematica dell'Università di Roma 1, I-00100 Roma, Italy and
Istituto Nazionale di Fisica Nucleare – Sezione di Roma 1, I-00100 Roma, Italy

Abstract. We prove that the renormalization program discussed in [1] can also be developed beyond the $\bar{\alpha}^2 = 2\pi(\sqrt{17}-1)$ threshold found in the preceding work. This result, as a byproduct, also allows a simplification in the technical part of the proof of ultraviolet stability in the φ_3^4-theory [2]. In the last section of this work we discuss, heuristically, but in some detail the interpretation of the sine-Gordon theory as a two-dimensional Yukawa gas for $\beta e^2 = \alpha^2 > 4\pi$.

Introduction

The two-dimensional sine-Gordon theory has been studied by Fröhlich [3] for the values of $\alpha^2 < 4\pi$ and, in the finite volume, by Benfatto et al. [1] for the values $\geqq 4\pi$. There it was proven that, for $\alpha^2 \geqq 4\pi$, the theory has to be renormalized; a renormalization procedure was constructed which amounted to subtracting from the potential $V_{0,I}^{(N)} = 2\lambda \int_I :\cos\alpha\varphi_\xi^{(N)}: d\xi$ some constant counterterms $C_i(N)$, whose number increases each time α^2 overcomes the thresholds $\alpha_{2n}^2 = 8\pi(1-1/2n)$ and which, of course, become infinite as the cutoff is removed ($N \to \infty$). Although the procedure envisaged in [1] seemed to prove the ultraviolet stability of the theory for any values $\alpha^2 < 8\pi$, some technical difficulties did not allow us to prove the upper bound of the ultraviolet stability for $\alpha^2 \geqq \bar{\alpha}^2 = 2\pi(\sqrt{17}-1)$.

The main goal of this paper is to prove that this spurious threshold can be removed. This is obtained by proving a theorem which allows us not to use the second part of Lemma 2 in [1], which was true only for $\alpha^2 < \bar{\alpha}^2$. As this lemma was also used in the proof of the upper bound in the φ_3^4-theory [2] and as this result can be immediately translated for that field theory model, this amounts to a slightly technical simplification of that proof also. Moreover now the proofs of the upper and lower bound appear more symmetric. The theorem is applied to prove explicitly the ultraviolet stability for all the values of $\alpha^2 < \frac{32}{5}\pi$ ($\frac{32}{5}\pi > \bar{\alpha}^2$), but the structure of this result and of the renormalization technique discussed in [1], allows us to conclude that the proof of stability for all $\alpha^2 < 8\pi$ is only a matter of