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Expansions and Phase Transitions for the Ground State of Quantum Ising Lattice Systems

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Abstract. Expansions for the ground state of some transverse Ising-like models are developed. These expansions are easily estimated by the solutions to some simple implicit equations. Short range or long range order obtains, depending on the coupling constants of the models.

Introduction

Let $H_\Lambda(\boldsymbol{\varepsilon})$ be a quantum lattice Hamiltonian associated with a finite volume $\Lambda\subset\mathbb{Z}^\nu$ and analytically dependent on coupling constants $\boldsymbol{\varepsilon}=(\varepsilon_1,\ldots,\varepsilon_n)$. Let $\psi_\Lambda(\boldsymbol{\varepsilon})$ be the corresponding normalized ground state eigenvector, and formally let

$$\varrho(\cdot\,;\boldsymbol{\varepsilon})=\lim_{\Lambda\uparrow\mathbb{Z}^\nu}\langle\psi_\Lambda(\boldsymbol{\varepsilon}),(\cdot)\,\psi_\Lambda(\boldsymbol{\varepsilon})\rangle \tag{1.1}$$

be the corresponding state defined by the $\psi_\Lambda(\boldsymbol{\varepsilon})$'s in the thermodynamic limit. The purpose of this article is to show that $\varrho(\boldsymbol{\varepsilon})$, as a function of $\boldsymbol{\varepsilon}$, can exhibit short or long range order, i.e. that $\varrho(\boldsymbol{\varepsilon})$ undergoes a phase transition in $\boldsymbol{\varepsilon}$ in all dimensions $\nu\geqq 1$, at least for a particularly simple family of Hamiltonians $H_\Lambda(\boldsymbol{\varepsilon})$.

The Hamiltonians we consider, transverse Ising models, which we assume to depend on two parameters $\boldsymbol{\varepsilon}=(\varepsilon,\delta)$, are defined by

$$-H_\Lambda(\varepsilon,\delta)=\sum_{i\in\Lambda}(1+\delta K(i,\sigma^z))\,\sigma^x(i)+\varepsilon\sum_{A\subset\Lambda}\hat{V}(A)\,\sigma^z(A)\,. \tag{1.2}$$

Here, $\sigma^x(i)$ and $\sigma^z(i)$ are the usual Pauli spin matrices acting at the site i,

$$\sigma^x(i)=\begin{pmatrix}0&1\\1&0\end{pmatrix},\qquad \sigma^z(i)=\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \tag{1.3}$$

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