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On $\overline{GL(4, \mathbb{R})}$-Covariant Extensions of the Dirac Equation*

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Abstract. Infinite component generalizations of both massless and massive Dirac equations are constructed which are covariant with respect to the double covering of the general linear group in four dimensions. These generalized Dirac equations can be made covariant with respect to the full diffeomorphism group of the spacetime manifold by replacing ordinary derivatives by covariant derivatives in the usual way.

1. Introduction

When ψ is a Dirac field transforming according to the complex four-dimensional spinor representation $a \to S(a)$ of the covering group $SL(2, \mathbb{C})$ of the connected (restricted) Lorentz group $SO_0(3,1)$, then by

$$(D(a)\psi)(x) := S(a)\psi(\theta(a)^{-1}x)$$

one defines a representation of $SL(2, \mathbb{C})$ in the space of Dirac fields; here θ is the real vector representation of $SL(2, \mathbb{C})$. The Dirac operator $i\gamma^\nu\partial_\nu + m$ commutes with the representation D [we are used to saying that the Dirac equation $(i\gamma^\nu\partial_\nu + m)\psi = 0$ is covariant with respect to Lorentz transformations].

A natural question to ask in a general relativistic contex is whether the representation S of $SL(2, \mathbb{C})$ could be extended to the group $G = \overline{GL(4, \mathbb{R})}$ [universal covering of the general linear group $GL(4, \mathbb{R})$] in such a way that the Dirac equation would be G-covariant. What would then be the generalized Dirac matrices? Assuming that the extension is carried out, the Dirac equation will be covariant with respect to an arbitrary diffeomorphism J in $(\mathbb{R}^4, g)$ (g is a Lorentz metric). One has only to replace ∂_ν by the space-time covariant derivative ∇_ν,

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