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General Structure of Nonlinear Evolution Equations in 1 + 2 Dimensions Integrable by The Two-Dimensional Gelfand–Dickey–Zakharov–Shabat Spectral Problem and their Transformation Properties

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Abstract. The general form of nonlinear evolution equations connected with the matrix two-dimensional Gelfand–Dickey–Zakharov–Shabat spectral problem is found. The infinite-dimensional abelian group of general Bäcklund transformations and infinite-dimensional abelian symmetry group for these equations are constructed.

I. Introduction

One of the main problems of the inverse scattering transform (IST) method is the description of equations integrable by this method (see e.g. [1, 2]). All the equations to which the IST method is applicable form the classes of the equations integrable by the same spectral problem. A very convenient and simple description of the partial differential equations integrable by the spectral problem

$$\frac{\partial \Psi}{\partial x} = \lambda A \Psi + P(x, t)\Psi \tag{1.0}$$

of the second matrix order has been given by Ablowitz, Kaup, Newell and Segur in [3]. Then this approach (AKNS approach) has been generalised to the problem (1.0) of an arbitrary order [4–10], to some other spectral problems [11, 12] and, in particular, to the one-dimensional Gelfand–Dickey spectral problem [13].

Recently the two-dimensional generalisation of the AKNS technique has been developed [14]. Namely, the two-dimensional arbitrary order spectral problem $\partial\Psi/\partial x + A(\partial\Psi/\partial y) + P(x, y, t)\Psi = 0$, where A is any diagonalisable constant matrix was considered: the general form of the nonlinear equations integrable by this problem and their Bäcklund transformations were found [14].

In the present paper we consider the two-dimensional matrix Gelfand–Dickey–