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# Localization in $\nu$-Dimensional Incommensurate Structures

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**Abstract.** We exhibit a class of quasi-periodic unbounded potential in the $\nu$-dimensional discrete Schrödinger equation, for which the spectrum is only pure point, with exponentially localized states and a dense set of eigenvalues in $\mathbb{R}$.

## Introduction

In a recent paper Fishman et al. [7] gave a solution of the Schrödinger equation

$$\Psi(n+1)+\Psi(n-1)+\lambda\tan\pi(x-n\omega)\Psi(n)=E\Psi(n), \qquad (1)$$

where $\omega$ is an irrational number, $x\in\mathbb{R}$ and $\lambda>0$. Actually, provided $\omega$ satisfies a diophantine condition, they gave an explicit expression of the eigenfunctions which turns out to decrease exponentially, and an implicit expression for the corresponding eigenvalues, leading to a dense set in the spectrum.

This result was interesting both because the solution was complete and also because there is no continuous part in the spectrum. Examples of discrete non-self-adjoint operators with only pure point spectrum were already known by Sarnak [11]. However in the self-adjoint case, with a bounded quasi-periodic potential we expect in general that aside from the pure point part there is a singular continuous spectrum. This seems to be the case for the almost Mathieu equation:

$$\Psi(n+1)+\Psi(n-1)+2\lambda\cos2\pi(x-n\omega)\Psi(n)=E\Psi(n), \qquad (2)$$

where it has been proved by Bellissard et al. [2] that if $\lambda$ is big enough, and $\omega$ satisfies a diophantine condition

$$\forall n,m\in\mathbb{Z}, \quad n\neq0, \quad |n\omega+m|\geqq\frac{\gamma}{|n|^{\sigma}}, \quad \sigma>1, \qquad (3)$$

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