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Examples of Discrete Schrödinger Operators with Pure Point Spectrum

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Abstract. We present a general approach for constructing potentials for the discrete Schrödinger equation of arbitrary dimension having only pure point spectrum. We give examples of limit periodic potentials of that kind such that the pure point spectrum is dense in an interval or a Cantor set of measure zero.

0. Introduction

In this note we consider the discrete Schrödinger operator

$$(Hu)_i = \varepsilon \sum_{|\ell|=1} u_{i+\ell} + d_i u_i, \qquad i \in \mathbb{Z}^m$$

with a small positive coupling constant ε acting on ℓ^2-sequences $u=(u_i)$ of arbitrary dimension $m \geqq 1$. Various examples of potentials $d=(d_i)$ are now known such that this operator has only pure point spectrum and a complete set of exponentially localized eigenvectors. It is our aim to present a general approach to the construction of these examples as well as some new ones. Namely, we construct *limit periodic* potentials d such that the pure point spectrum is dense in $[0,1]$ in one case, and dense in a Cantor set of measure zero in another case.

We proceed as follows. We write

$$H = D + \varepsilon \Delta$$

in the form of a $2m$-dimensional Jacobi matrix, with $D = \operatorname{diag}(d)$ and

$$\Delta = \sum_{|\ell|=1} (\delta_{i,j+\ell}),$$

where $|\ell| = \sum_{\mu=1}^{m} |\ell_\mu|$ for $\ell = (\ell_1, \dots, \ell_m)$. For a suitably fixed D and sufficiently small ε, we then prove the existence of another diagonal matrix $\hat{D}$ close to D such that

$$V^{-1}(\hat{D} + \varepsilon\Delta)V = D, \tag{0.1}$$