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$(\text{Higgs})_{2,3}$ Quantum Fields in a Finite Volume

III. Renormalization

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Abstract. This is the third paper of a series, and contains a proof of the bounds on the effective actions needed in the two previous papers. The proof is based on perturbative analysis of renormalization.

1. Introduction. Formulation of a Basic Estimate

In this paper we will prove the basic properties of the perturbation expansions used in the two previous papers [1, 2]. These properties form a very important part of the proof of ultraviolet stability. We will prove them for the considered model, but the method extends in a natural way to more complicated models. The characteristic feature of the method is that it almost does not use a momentum representation for the expression in the perturbation expansion. It is based entirely on real space properties of propagators. These properties hold for propagators in more complicated theories also (e.g. non-abelian gauge theories, fermion field theories) and it is the reason for these natural extensions.

In this paper we use notation, results and formulas of the two previous papers [1, 2] and we will refer to them adding I or II before the numberings of the corresponding papers. Many properties of perturbative expressions were used in [1, 2] and it is difficult to formulate them in the form of separate theorems; the formulations would be very long. Instead we will describe and analyze a general expression and we will prove for it some basic estimate from which all the necessary properties will follow easily. Let us recall the formula for interaction terms of the action of the model after k renormalization transformations and let us write it in the form appearing in the proof of the upper bound. We will write this formula rescaled to the unit lattice with respect to "new" field variables after k steps. The torus T_η is replaced by the corresponding subset $B^k(\Lambda_2^{(k-1)\prime})$, which will be denoted by Ω. The set $\Lambda_2^{(k-1)\prime}$ is a sum of big blocks, so the assumptions of Propositions I.2.1–I.2.3 are satisfied for Ω. The "old" vector field can be represent-

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