On the Structure of Space–Time Caustics†

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Abstract. Caustics formed by timelike and null geodesics in a space–time M are investigated. Care is taken to distinguish the conjugate points in the tangent space (T-conjugate points) from conjugate points in the manifold (M-conjugate points). It is shown that most nonspacelike conjugate points are regular, i.e. with all neighbouring conjugate points having the same degree of degeneracy. The regular timelike T-conjugate locus is shown to be a smooth 3-dimensional submanifold of the tangent space. Analogously, the regular null T-conjugate locus is shown to be a smooth 2-dimensional submanifold of the light cone in the tangent space. The smoothness properties of the null caustic are used to show that if an observer sees focusing in all directions, then there will necessarily be a cusp in the caustic. If, in addition, all the null conjugate points have maximal degree of degeneracy (as in the closed Friedmann–Robertson–Walker universes), then the space-time is closed.

1. Introduction

Gravitational focusing plays an important role in general relativity both observationally through the discovery of the gravitational lens effect (Walsh et al. 1979 [1]) and theoretically in the proofs of the singularity theorems (Hawking and Ellis [2]). A non-uniform gravitational field gives rise to tidal forces which tend to have a converging effect on a bundle of light rays. It is the attractive nature of the gravitational force which causes a bundle of rays to converge and focus rather than diverge. An observational effect of focusing is that objects are magnified as in a lens. In general, the magnification depends on the transverse direction. This means that images will be distorted; for example, a circular galaxy may appear elliptical.

The points where geodesics refocus are called conjugate points [2, Chapt. 4]. The geometric locus of such points, the conjugate locus, is said to be a caustic. The proofs of the singularity theorems depend on the existence of a pair of conjugate points

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