Commun. Math. Phys. 88, 327–338 (1983)

Communications in Mathematical Physics

A Local Time Approach to the Self-Intersections of Brownian Paths in Space

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Abstract. We study the Brownian functional

$$\alpha(x,B) = \iint_B \delta_x(W_t - W_s)\,ds\,dt,$$

where W_t is a Brownian path in two or three dimensions. For B off the diagonal we identify $\alpha(x, B)$ with a local time, and establish the Hölder continuity of $\alpha(x, B)$ in both x and B.

1. Introduction

A classical theorem of Dvoretzky, Erdös and Kakutani [1950] states that a Brownian path W_t will intersect itself in both two and three dimensions. This fact is at the heart of Symanzik's approach to Euclidean quantum field theory [1969] where the key role is played by the purely formal expression

$$\int_0^h \int_0^h \delta_x(W_t - W_s)\,ds\,dt. \tag{1.1}$$

Here δ_x is the Dirac delta function concentrated at x. When $x = 0$, (1.1) is meant as a measure of the amount of time t, $0 \leqq t \leqq h$, spent by the path in intersecting itself. This expression also appears in the study of polymers, see Edwards [1965] and Westwater. In this paper we employ the general perspective of the theory of local times to analyze (1.1). For an excellent overview of local times, together with extensive references, we refer to the survey paper of Geman and Horowitz [1980].

The only general method for studying local times of Gaussian processes involves local non-determinism (LND), a concept introduced by Berman [1973], and generalized by Pitt [1978] and Cuzick. Since the process underlying (1.1) does not appear to be locally nondeterministic, we are forced to develop a new approach. We refer the reader to Rosen [1981], and Geman, Horowitz and Rosen [1981], where

* Partially supported by NSF–MCS–80–02940