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Positive Mass Theorems for Black Holes

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Abstract. We extend Witten's proof of the positive mass theorem at spacelike infinity to show that the mass is positive for initial data on an asymptotically flat spatial hypersurface Σ which is regular outside an apparent horizon H. In addition, we prove that if a black hole has electromagnetic charge, then the mass is greater than the modulus of the charge. These results are also valid for the Bondi mass at null infinity. Finally, in the case of the Einstein equation with a negative cosmological constant, we show that a suitably defined mass is positive for data on an asymptotically anti-de Sitter surface Σ which is regular outside an apparent horizon.

1. Introduction

The gravitational potential energy of any system is always negative because gravity is an attractive force. In Newtonian theory one can shrink any system to an arbitrarily small size and make the total energy indefinitely negative. However it appears that one cannot do this according to the general theory of relativity. As one considers smaller and smaller configurations for the system, the potential energy becomes more negative but the total energy, i.e. the rest mass plus potential energy plus kinetic energy seems to remain positive. At a certain critical size an outer future trapped surface appears [1, 2]. This is a closed spacelike 2-surface which is in such a strong gravitational field that the outgoing future directed light rays or null geodesics orthogonal to it are converging, i.e. they are being dragged back by the gravitational field. The outer boundary of the region on a spacelike hypersurface which contains outer future trapped surfaces is called the future apparent horizon [2, 3]. By the singularity theorems (see [2]) the system must collapse to produce a spacetime singularity provided that certain physically reasonable conditions hold. According to the unproved but very plausible cosmic

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