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# Irreducible Kernels and Nonperturbative Expansions in a Theory with Pure $m \to m$ Interaction

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**Abstract.** Recent results on the structure of the $S$ matrix at the $m$-particle threshold ($m \geqq 2$) in a simplified $m \to m$ scattering theory with no subchannel interaction are extended to the Green function $F$ on the basis of off-shell unitarity, through an adequate mathematical extension of some results of Fredholm theory: local two-sheeted or infinite-sheeted structure of $F$ around $s=(m\mu)^2$ depending on the parity of $(m-1)(v-1)$ (where $\mu>0$ is the mass and $v$ is the dimension of space-time), off-shell definition of the irreducible kernel $U$ which is the analogue of the $K$ matrix in the two different parity cases $(m-1)(v-1)$ odd or even, and related local expansion of $F$, for $(m-1)(v-1)$ even, in powers of $\sigma^\beta \ln\sigma(\sigma=(m\mu)^2-s)$. It is shown that each term in this expansion is the dominant contribution to a Feynman-type integral in which each vertex is a kernel $U$. The links between the kernel $U$ and Bethe-Salpeter type kernels $G$ of the theory are exhibited in both parity cases, as also the links between the above expansion of $F$ and local expansions, in the Bethe-Salpeter type framework, of $F_\lambda$ in terms of Feynman-type integrals in which each vertex is a kernel $G$ and which include both dominant and subdominant contributions.

## 1. Introduction

The two-sheeted, square-root type structure of the $S$ matrix at the two-particle threshold of a $2 \to 2$ process in space-time dimension $v=4$ is an old result of the sixties [1–4] derived from two-body unitarity. The result has also been extended to the Green function $F$ of the process (whose mass shell restriction is the scattering function $T$) either [5] in the Bethe-Salpeter framework in which the Bethe-Salpeter kernel $G$ is assumed to be irreducible (i.e. analytic at thereshold) or [4] on the basis of off-shell unitarity (= asymptotic completeness).

Further results on the structure of the multiparticle $S$ matrix and Green functions near other Landau singularities have been obtained in axiomatic $S$ matrix [6–8] and field theory [4, 9, 10]. They include in particular local decompositions in terms of "Feynman-type" contributions, or of analogous