Commun. Math. Phys. 88, 207–234 (1983)

Communications in Mathematical Physics

A Metal-Insulator Transition for the Almost Mathieu Model

J. Bellissard*, R. Lima, and D. Testard**

Centre de Physique Théorique, CNRS, F-13288 Marseille Cedex 9, France

Abstract. We study the spectrum of the almost Mathieu hamiltonian:

$$(H_x\psi)(n) = \psi(n+1) + \psi(n-1) + 2\mu\cos(x - n\theta)\psi(n), \qquad n\in\mathbb{Z},$$

where θ is an irrational number and x is in the circle $\mathbb{T}$. For a small enough coupling constant μ and any x there is a closed energy set of non-zero measure in the absolutely continuous spectrum of H. For sufficiently high μ and almost all x we prove the existence of a set of eigenvalues whose closure has positive measure. The two results are obtained for a subset of irrational numbers θ of full Lebesgue measure.

I. Introduction

The aim of this paper is to study some properties of the spectrum of operators of the form:

$$H_x^{(\mu)}\psi(n) = \psi(n+1) + \psi(n-1) + \mu V(x - n\theta)\psi(n), \tag{1.1}$$

where $\psi\in\ell^2(\mathbb{Z})$, V is a continuous function on the circle $\mathbb{T}$, θ is an irrational number, $x\in\mathbb{T}$ and μ is a real positive number (the coupling constant). From the physical point of view, both the dependence of the spectrum on μ, as well as the growth of the eigenfunctions as $n\to\infty$ are crucial.

The first example of the treatment of an almost periodic potential goes back to Peierls [21] where the Schrödinger operator defined in (1.1) describes the one band hamiltonian for a Bloch electron in a magnetic field, in the approximation where the interband contributions is neglected; see also [22]. The prediction of Little

* Also at: Université de Provence

** Also at: Centre Universitaire d'Avignon