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Absence of Diffusion in the Anderson Tight Binding Model for Large Disorder or Low Energy

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Abstract. We prove that the Green's function of the Anderson tight binding Hamiltonian decays exponentially fast at long distances on $\mathbb{Z}^\nu$, with probability 1. We must assume that either the disorder is large or the energy is sufficiently low. Our proof is based on perturbation theory about an infinite sequence of block Hamiltonians and is related to KAM methods.

1. Introduction

1.1. General Background

In this paper we analyze the Schrödinger operator (Hamiltonian)

$$H = H(v) = -\Delta + v\,, \tag{1.1}$$

where Δ is the finite difference Laplacian on $\mathbb{Z}^\nu$, and $v = \{v(j)\}$ is a random potential. We shall consider the case in which the $v(j)$ are independent (e.g. Gaussian) random variables with mean 0 and variance γ. The Hamiltonian (1.1) was introduced by Anderson [1] to model the dynamics of a quantum mechanical particle – the electron – moving in a random medium. The random medium may be thought of as a crystal with impurities of random strength $v(j)$. The variance γ measures the overall strength of the impurities or of the disorder. In this model the electron interacts only with the impurities. Electron-electron interactions and thermal effects are neglected.

Let $\psi_t = e^{itH}\psi_0$ be the time evolution of a wave function ψ_0 supported near the origin, e.g. $\psi_0(j) = \mathrm{const}\, e^{-|j|}$. In order to describe the long time behaviour of our particle, consider the spread of ψ_t as measured by

$$r^2(t) \equiv \sum_{x \in \mathbb{Z}^\nu} x^2 |\psi_t(x)|^2\,. \tag{1.2}$$

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