

## $C^*$ -Algebras and Automorphism Groups

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**Abstract.** Let  $(A, G, \alpha)$  be a  $C^*$ -dynamical system with  $G$  a topological group. Let  $\pi$  be a representation of  $A$ . We will show that there exists a quasi-equivalent representation  $\hat{\pi}$  to  $\pi$  which is a covariant representation, if and only if the folium of  $\pi$  is invariant under the action of  $G$  and this action is strongly continuous.

### I. Introduction and Notation

Symmetries are one of the most powerful concepts in physics. Many of the classification schemes of physical objects are based on symmetry-groups. Therefore it is no wonder that one finds a vast amount of literature on this subject. In earlier times most of these investigations focused on the classification of group representations. In the last  $1\frac{1}{2}$  decade, however, the interest has changed more to the investigation of the interplay between the symmetry-group and the algebra of observables or the field-algebra. This subject now goes under the name of  $C^*$ -dynamical systems. Although this name is unsatisfactory from the physical point of view, because usually only the kinematical possibilities are studied, we use this name in this paper. The main tool of this subject is what has been called by Doplicher et al. [8] the covariance-algebra and which is now known as the crossed product between a  $C^*$ -algebra and a group. For a good survey on this subject see the book of Pedersen [11], where one also finds a list of references. One of the objects of this theory is to characterize the representations in which we have also a continuous representation of the symmetry-group implementing the automorphisms. This problem has been answered modulo problems of multiplicity by the author [3, 4].

Looking at this part of the theory of  $C^*$ -dynamical systems, one finds that there are two assumptions which are unsatisfactory. The first assumption is the continuity assumption, which says that the expressions  $g \rightarrow \alpha_g(x)$  have to be continuous functions on the group with values in the  $C^*$ -algebra furnished with the norm topology. However, in quantum field theory or statistical mechanics one