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Renormalization Group for a Critical Lattice Model

Effective Interactions Beyond the Perturbation Expansion or Bounded Spins Approximation

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Abstract. New methods are developed for the study of the Kadanoff-Wilson renormalization group for critical lattice systems of unbounded spins. The methods are based on a combination of expansion and analyticity techniques and are applied to a nonlocal hierarchical model of the dipole gas. They remove the main obstacle against the use of the block spin strategy in more realistic models such as ϕ_d^4, the dipole gas and the anharmonic crystal.

1. Introduction

In the previous publications [4–6] the present authors have developed methods for a non-perturbative analysis of the Kadanoff-Wilson renormalization group (RG) in the context of massless lattice theories such as the dipole gas and ϕ_d^4, $d \geqq 4$, at the critical point. It was soon realized that the main problem was the simultaneous control of the clustering and positivity properties of the effective interactions. To study these questions separately, we introduced a hierarchical approximation to the systems (see [4] for more details and the motivation). We considered different cases depending on whether the fields describing fluctuations on a given distance scale were bounded or unbounded in magnitude and whether they had covariance totally local or one with an exponential falloff (as in the real models). The bounded nonlocal model was suitable for the study of clustering [4, 5] whereas the unbounded local model dealt with the positivity problem [6]. The first case was solved by use of a cluster expansion to compute the effective potential, the solution of the second one was based on the study of the analyticity improving properties of the RG transform.

In the present paper we combine these two ideas to carry out the analysis in the unbounded nonlocal case. We prove that for a wide class of potentials the RG

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