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Supersymmetric Two-Dimensional Toda Lattice

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Abstract. The two-dimensional Toda lattice connected with contragradient Lie superalgebras is studied. The systems of linear equations associated with the models for which the inverse scattering method is applicable are written down. The reduction group is calculated.

1. Introduction

Many papers in the last few years considerable discuss the hidden symmetries of integrable systems. Bogoyavlensky discovered that the classical Toda lattice (TL) is connected with the simple Lie algebras [1]. Then Leznov and Saveliev showed [2] that the periodic TL corresponds to contragradient Lie algebras (Kac-Moody algebras). This connection enables investigation of the systems by means of the Inverse Scattering Method (ISM) (see also [3]). On the other hand the simplest cases of the two-dimensional Toda lattice (TTL) – the Sinh-Gordon equation and the Liouville equation have supersymmetric extensions, while the integration by the ISM is applicable as before [4, 5]. In the present work we construct a supersymmetric version of TTL and discover the connection of these systems with the Contragradient Lie Superalgebras (CLS) classified by Kac [7]. This connection allows us to write down the equations of the systems as "the zero-curvature conditions" (Zakharov-Shabat equations) and, in principle, to apply the ISM.

The systems under consideration are described by the action

$$S=\int d^2x d\theta_1 d\theta_2\left(-i\sum_{j=1}^{h}\Phi^j D_1 D_2 \Phi^j - U(\Phi)\right), \tag{1.1}$$

where $x=(x_0, x_1)$, θ_1, θ_2 are scalar, and the Grassmann superspace parameters, D_1 and D_2, are supersymmetric covariant derivatives, and $\Phi=(\Phi^1, \ldots, \Phi^n)$ is a multiplet of the scalar bosonic superfields.