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# Hamiltonian Structures and Lax Equations Generated by Matrix Differential Operators with Polynomial Dependence on a Parameter

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**Abstract.** We investigate a general set of equations which can be studied by the inverse scattering method.

In the present paper we study Hamiltonian structures and equations of the Lax type which arise in connection with a first-order linear differential operator $L=\partial+U_0+U_1\zeta+\ldots+U_n\zeta^n+A\zeta^{n+1}$, where $\partial=\frac{d}{dx}$; $U_i$ are matrices, and $A$ is a constant diagonal matrix. The case $n=0$ is well-known.

Much attention is also paid to the Lagrangian formalism for these equations and its connection with the Hamiltonian approach. Two different Lagrangians are found.

## 1. Lie Algebra $R_-$

$\mathscr{A}$ is a differential algebra consisting of the polynomials (with real coefficients) in matrix elements of some matrices $U_0, U_1, \ldots, U_n$ and their derivatives with respect to $x$ of arbitrary orders. Thus elements of $U_i$ are independent generators of this differential algebra: $R_-$ is a Lie algebra consisting of the matrix bundles, $X=\sum_{i=0}^{n} X_i\zeta^{-i-1}$, where elements of $X_i$ belong to $\mathscr{A}$, and $\zeta$ is a formal parameter. In this algebra the commutator is introduced by

$$[X, Y]_{\mathfrak{z}}=\overline{[X, Y](\zeta^{n+1}+\mathfrak{z})}. \tag{1.1}$$

Here [ , ] is the ordinary commutator of matrices, $\mathfrak{z}$ is a fixed real number, the bar symbolizes cutting out a segment of a series in $\zeta^{-1}$ from $\zeta^{-1}$ to $\zeta^{-n-1}$.

It is clear that the commutator (1.1) is a linear combination of two commutators: from $[X, Y]$ we cut out either the second half of the powers of $\zeta^{-1}$ (from $\zeta^{-n-2}$ to $\zeta^{-2n-2}$) and multiply it by $\zeta^{n+1}$, or the first half (from $\zeta^{-1}$ to $\zeta^{-n-1}$). So we have a family of commutators depending on the parameter $\mathfrak{z}$.