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Rigorous Construction of Planar Diagram Field Theories in Four Dimensional Euclidean Space

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Abstract. Asymptotically free quantum field theories with planar Feynman diagrams [such as SU(∞) gauge theory] are considered in 4 dimensional Euclidean space. It is shown that if all particles involved have non-vanishing masses and if the coupling constant(s) λ (or g^2) are small enough ($\lambda \leqq \lambda^{\text{crit}}$), then an absolutely convergent procedure exists to obtain Green functions that uniquely solve the Dyson-Schwinger equations.

1. Introduction

In two previous papers [1, 2] this author explained his interest in planar diagram field theories. To date no analytic method for summing planar diagrams is known, although interesting exact features of the corresponding $N \to \infty$ field theories were displayed [3]. We now suspect that convergent calculational procedures do exist in these theories, but we are unable to prove this for the most interesting case: SU(∞) QCD. The problem is the fact that the gluons are massless. In this paper we consider a closely related set of theories. The Feynman rules may be as in planar QCD, except that we take all particles involved to be massive, and the coupling constant must be smaller than a certain limit. Furthermore, our theory must be asymptotically free. Since we only look at the Feynman rules and are not concerned about unitarity or positivity of the energy, the signs of the couplings can always easily be arranged in such a way that asymptotic freedom is possible. For such a theory we will construct an absolutely convergent calculational procedure. In another publication [4] we already indicated how this may imply absolute Borel summability of the perturbation expansion of this system. Since the mass of the particles will only be needed in the very end of our argument we will also be able to draw certain conclusions about the Borel functions in massless SU(∞) QCD. Further details are postponed to a future publication. There may be some optimism that SU(∞) QCD may also be rigorously constructed sometime, but prospects for finite SU(N) theories seem to be much more remote.

Our technique will be that of [2]. There it was shown that if we sum only the ultraviolet-convergent planar diagrams then this sum has a finite radius of