Some Remarks on BRS Transformations, Anomalies and the Cohomology of the Lie Algebra of the Group of Gauge Transformations

L. Bonora[*,1] and P. Cotta-Ramusino[**,2]

1 Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
2 Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Abstract. We show that ghosts in gauge theories can be interpreted as Maurer-Cartan forms in the infinite dimensional group $\mathscr{G}$ of gauge transformations. We examine the cohomology of the Lie algebra of $\mathscr{G}$ and identify the coboundary operator with the BRS operator. We describe the anomalous terms encountered in the renormalization of gauge theories (triangle anomalies) as elements of these cohomology groups.

1. Introduction

A few years ago Becchi et al. [1, 2] noticed that in gauge theories the effective Lagrangian (with gauge breaking and ghost terms included) is invariant under a special class of transformations (BRS transformations). The ensuing Ward-Takahashi identities allow an elegant proof of renormalizability and the nilpotent nature of the BRS transformations is apparently the key for the unitarity of the S-matrix.

   More recently it was discovered [3] that many effective Lagrangians in gauge theories are invariant also under a second set of transformations, which we call anti-BRS transformations.

   BRS transformations have attracted much interest ever since (see for instance [4] or [5]). There have been also some attempts to give a geometrical interpretation of the BRS transformations and the ghost fields. In [6] the ghost fields are viewed essentially as the "vertical part" of a connection form (locally defined). Reference [7] gives a modification of the ideas of [6] and attempts to give a geometrical interpretation of the antighosts as well. However, both attempts exhibit unsatisfactory aspects.