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Thermodynamics of Black Holes in Anti-de Sitter Space

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Abstract. The Einstein equations with a negative cosmological constant admit black hole solutions which are asymptotic to anti-de Sitter space. Like black holes in asymptotically flat space, these solutions have thermodynamic properties including a characteristic temperature and an intrinsic entropy equal to one quarter of the area of the event horizon in Planck units. There are however some important differences from the asymptotically flat case. A black hole in anti-de Sitter space has a minimum temperature which occurs when its size is of the order of the characteristic radius of the anti-de Sitter space. For larger black holes the red-shifted temperature measured at infinity is greater. This means that such black holes have positive specific heat and can be in stable equilibrium with thermal radiation at a fixed temperature. It also implies that the canonical ensemble exists for asymptotically anti-de Sitter space, unlike the case for asymptotically flat space. One can also consider the microcanonical ensemble. One can avoid the problem that arises in asymptotically flat space of having to put the system in a box with unphysical perfectly reflecting walls because the gravitational potential of anti-de Sitter space acts as a box of finite volume.

1. Introduction

The first indication that black holes have thermodynamic properties came with the discovery that in the classical theory of general relativity the area of the event horizon [1] (or equivalently, the square of the irreducible mass [2]) never decreases. There is an obvious analogy with the second law of thermodynamics with the area of the event horizon playing the role of entropy. There were also analogies to the zeroth and first laws of thermodynamics in which the role of temperature was played by a quantity called the surface gravity κ which measured the strength of the gravitational field at the event horizon [3]. These similarities led Bekenstein [4] to suggest that some multiple of the area of the event horizon,

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