Commun. Math. Phys. 87, 497–504 (1983)

Communications in Mathematical Physics

Non-Translation-Invariant States in Two Dimensions

S. B. Shlosman

Institute for Problems of Information Transmission, Moscow, USSR

Abstract. We construct a class of models with translation-invariant interaction for which in dimension two there already exist non-periodic Gibbs states at low temperatures.

1. Introduction

The hypothesis about absence of non-translation-invariant states in the two-dimensional Ising model was one of the most long-standing problems. The first major development in this direction was made by Gallavotti [1], who proved, by means of virial expansion, that at low temperature the $(\pm)$-boundary condition leads to a translation-invariant state. He showed that the separation line between $(+)$-phase and $(-)$-phase in a volume V with such a boundary condition fluctuates at non-zero temperature with a fluctuation of order $\sqrt[4]{V}$. Afterwards, several papers were published on this subject, until Aizenman [2] proved the validity of the above hypothesis. This result can be stated in the following way: All non-periodic ground states of the two-dimensional Ising model are unstable.

In 1980, R. L. Dobrushin conjectured that the same instability holds (at low temperatures, at least) for any two-dimensional model whose hamiltonian has only finitely many periodic ground states (pure phases) and satisfies the Peierls condition [3]: i.e., the creation of an "island" D of one pure phase in a "sea" of another pure phase leads to an increase of energy which is greater than $\varrho\cdot|\partial\mathscr{D}|$, where ϱ is some positive constant (the "Peierls constant"), and $|\partial\mathscr{D}|$ is the length of the boundary of $\mathscr{D}$.

While the proof of this conjecture is still in progress, it was surprising to find a class of models which possesses both periodic and non-periodic Gibbs states at a cost of having infinitely many periodic ground states. The Peierls condition is still satisfied. The underlying non-translation invariant ground states are of the following "stair" structure:

$$\varphi_{(t_1,t_2)}=nt_1\,,\quad (t_1,t_2)\in\mathbb{Z}^2\,,\quad n\in\mathbb{Z}^1,\quad n\neq 0\,.$$

0010-3616/83/0087/0497/$01.60