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From the Euclidean Group to the Poincaré Group via Osterwalder–Schrader Positivity

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Abstract. Given a continuous representation of the Euclidean group in $n+1$ dimensions, together with a covariant system of subspaces, which satisfies Osterwalder–Schrader positivity, we construct a continuous unitary representation of the orthochronous Poincaré group in $n+1$ dimensions satisfying the spectral condition. A similar result holds for the covering groups of the Euclidean and Poincaré group.

Osterwalder–Schrader positivity allows the analytic continuation of a theory in imaginary time to a quantum theory in real time (Osterwalder and Schrader [6], Klein and Landau [3], Glimm and Jaffe [1]).

When this analytic continuation transforms a field theory covariant with respect to the Euclidean group into one covariant with respect to the Poincaré group, the procedure has been to analytically continue the Schwinger functions of the Euclidean theory, which are distributions invariant under the action of the Euclidean group, into other distributions called Wightman functions, which are then shown to be invariant under the action of the Poincaré group by analytic continuation of the partial differential equations that express the Euclidean invariance of the Schwinger functions (Nelson [5], Osterwalder and Schrader [6]).

In this article we show how Osterwalder–Schrader positivity allows the construction of a unitary representation of the Poincaré group directly from a representation of the Euclidean group with a covariant system of subspaces. The new tool that makes that possible is our theory of symmetric local semigroups (Klein and Landau [4]; see also Fröhlich [10]).

We start by considering the analytic continuation of unitary representations of the Euclidean group. Next we prove similar results for the covering groups of the Euclidean and Poincaré groups. Finally, we describe the extension to representations of the Euclidean group (or its covering group) on topological vector

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