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Bounds on the Decay of Correlations for $\lambda(\nabla\phi)^4$ Models*

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Abstract. We consider models, with an abelian continuous group of symmetry, of the type:

$$H = \sum_x \left[\frac{1}{2}(\nabla_x\phi)^2 + \frac{\lambda}{4}(\nabla_x\phi)^4\right].$$

We generalize Brascamp-Lieb inequalities to get (λ-independent) bounds on the low momentum behaviour of general correlation functions when these are truncated into two clusters. We then use this result to derive an asymptotic expansion (up the second order in λ) of the dielectric constant of this system.

I. Introduction

In this paper we consider perturbations of the massless Gaussian lattice field which preserve its abelian continuous group of symmetry. Our results mainly concern the following model of an anharmonic crystal (defined on $\mathbb{Z}^d$)

$$H = 1/2\sum_x (\nabla_x\phi)^2 + \lambda/4(\nabla_x\phi)^4 .$$

The main particularity of this model is that its correlations, e.g. $\langle\nabla_0\phi\nabla_x\phi\rangle$, are nonintegrable ($d>1$). A lot of work is now being done in order to understand critical properties of classical lattice systems [1–6]. This model, one of the simplest nonexplicitly soluble critical models, has been investigated quite a lot. Two approaches have been developed so far. One is based on a rigorous version of renormalization group ideas and has been considered by [3]. The results produced mainly concern weak coupling (λ small). Another approach based on nonperturbative methods, producing therefore results which are not sensitive to the strength of λ, was proposed in [4–6]. In this note we want to develop further the second approach to get more detailed information about the decay of correlations.

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