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A Criterion of Integrability for Perturbed Nonresonant Harmonic Oscillators. "Wick Ordering" of the Perturbations in Classical Mechanics and Invariance of the Frequency Spectrum

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Abstract. We introduce an analogue to the renormalization theory (of quantum fields) into classical mechanics. We also find an integrability criterion guaranteeing the convergence of Birkhoff's series and an algorithm for modifying the hamiltonian to fix the frequency spectrum of the quasi-periodic motions. We point out its possible relevance to the transition to chaos.

1. Introduction, Notations and Results

In a famous paper [1] Poincaré proved the generic nonexistence of analytic prime integrals for systems which are obtained by perturbing an integrable system. This system is described in its action-angle variables $(\mathbf{A}, \boldsymbol{\varphi})$ by an analytic hamiltonian $h_0(\mathbf{A})$ such that

$$\operatorname{rank} \frac{\partial^2 h_0}{\partial \mathbf{A} \partial \mathbf{A}}(\mathbf{A}) \geqq 2, \tag{1.1}$$

where $\mathbf{A} = (A_1, \ldots, A_\ell)$ denote the ℓ action variables, $\boldsymbol{\varphi} = (\varphi_1, \ldots, \varphi_\ell)$ denote the ℓ conjugate angles, and we suppose that the system's phase space is

$$V \times \mathrm{T}^\ell, \tag{1.2}$$

where V is a closed sphere in R of radius r, fixed once and for all, and T^ℓ is the ℓ-dimensional torus. The number ℓ is the number of degrees of freedom.

The theorem of Poincaré deals with perturbations of the form

$$f_0(\mathbf{A}, \boldsymbol{\varphi}, \varepsilon) = \sum_{\gamma \in Z^\ell} f_{0\gamma}(\mathbf{A}, \varepsilon) e^{i\gamma \cdot \boldsymbol{\varphi}}, \tag{1.3}$$

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