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L^2-Exponential Lower Bounds to Solutions of the Schrödinger Equation

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Abstract. We study decay properties of solutions ψ of the Schrödinger equation $(-\Delta+V)\psi=E\psi$. Typical of our results is one which shows that if $V=o(|x|^{-1/2})$ at infinity or if V is a homogenous N-body potential (for example atomic or molecular), then if $E<0$ and $\alpha>\sqrt{-E}$, $e^{\alpha|x|}\psi\notin L^2(\mathbb{R}^n)$. We also construct examples to show that previous essential spectrum-dependent upper bounds can be far from optimal if ψ is not the ground state.

I. Introduction

In recent years there has been much interest in the asymptotic behavior of L^2-solutions to the Schrödinger equation

$$(-\Delta+V)\psi=E\psi. \tag{1.1}$$

By far, most of the effort has gone into proving upper bounds to solutions of (1.1) with E outside the essential spectrum of $-\Delta+V$. Recent work on this subject can be found in [1–3, 12, 19]. The results of Agmon [1, 2] for the N-body problem are the most general. Agmon shows that solutions ψ of (1.1) satisfy (under certain conditions)

$$|\psi(x)|\leqq C_\varepsilon \exp(-(1-\varepsilon)\varrho_E(x)) \tag{1.2}$$

for $\varepsilon>0$, where $\varrho_E(x)$ is (in principle) an explicitly computable function. This generalizes the earlier result in [25] which states that for N-body potentials

$$|\psi(x)|\leqq C_\varepsilon \exp(-(1-\varepsilon)\sqrt{\Sigma-E}|x|), \tag{1.3}$$

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