Commun. Math. Phys. 87, 229–252 (1982)

Communications in Mathematical Physics

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Infinite Differentiability for One-Dimensional Spin System with Long Range Random Interaction

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Abstract. We consider one-dimensional spin systems with Hamiltonian:

$$H(\sigma_\Lambda) = -\sum_{t,t' \in \Lambda} \frac{\varepsilon_{tt'}}{|t-t'|^\alpha} \sigma_t \sigma_{t'} - h \sum_{t \in \Lambda} \sigma_t,$$

where $\varepsilon_{tt'}$ are independent random variables and, using decimation and the cluster expansion, we show that, when $\alpha > 3/2$ and $\mathbb{E}(\varepsilon_{tt'}) = 0$, for any magnetic field h and inverse temperature β, the correlation functions and the free energy are C^∞ both in h and β.

Moreover we discuss an example, obtained by a particular choice of the probability distribution of the $\varepsilon_{tt'}$'s, where the quenched magnetization is C^∞ but fails to be analytic in h for suitable h and β.

1. Introduction and Results

We consider a one-dimensional system with random interaction enclosed in a box Λ whose energy, for a given spin configuration σ_Λ in Λ, is:

$$H(\sigma_\Lambda) = -\sum_{\substack{t_1, t_2 \in \Lambda \\ t_1 \neq t_2}} \frac{\varepsilon_{t_1 t_2}}{|t_1 - t_2|^\alpha} \sigma_{t_1} \sigma_{t_2} - h \sum_{t \in \Lambda} \sigma_t, \tag{1.1}$$

where $\sigma_t \in \{1, -1\}$, $3/2 < \alpha < 2$[1] and $\varepsilon_{t_1 t_2}$ are independent random variables defined in the probability space $(\Omega, \Sigma, \mathbb{P})$.

In the sequel we will consider the following conditions on the $\varepsilon_{t_1 t_2}$:

C1) $\mathbb{E}(\varepsilon_{t_1 t_2}) = 0$,

C2) $\exists \bar{\varepsilon} : |\varepsilon_{t_1 t_2}| < \bar{\varepsilon} \quad \forall t_1, t_2 \in \mathbb{Z}$,

C3) $\mathbb{E}(\varepsilon_{t_1 t_2}^2) \geqq a$ for some $a > 0$,

C4) the probability distribution of $\varepsilon_{t_1 t_2}$ depends only on $|t_1 - t_2|$ (translation invariance).

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1 For $\alpha > 2$ the stochastic character of the interaction is irrelevant (see [1] and Remark 3 of Sect. 4)

0010-3616/82/0087/0229/$04.80