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The Wightman Axioms for the Fermionic Federbush Model

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Abstract. All Wightman axioms, including asymptotic completeness, are proved for the Federbush model with coupling constants in the range $(-1/2, 1/2)$.

1. Introduction

The relativistic quantum field theory under consideration in this paper was invented and formally solved by Federbush in 1961 [1, 2]. Called the Federbush model ever since, it describes two species of one-dimensional charged massive fermions interacting through a current-pseudocurrent coupling. A few years later Wightman [3] studied (among other things) a number of field theories known to be formally soluble, and in particular the Federbush model, with the aim of fitting these theories into the framework of axiomatic quantum field theory. He observed that a one-dimensional massive free Dirac field has a current that is the gradient of a "pseudopotential" σ, and went on to show that σ is a local field that is not local with respect to the free field. He then indicated how the Federbush field operator might be given a rigorous meaning in terms of the object $\vdots\exp(i\pi\lambda\sigma)\vdots$. The triple dots denote vacuum subtractions, which are already necessary to ensure the object σ^n has a well-defined meaning. In subsequent unpublished work Challifour and Wightman [4] proved the field $\vdots\sigma^n\vdots$ is a local operator-valued tempered distribution, but they could not show this for the field $\vdots\exp(i\pi\lambda\sigma)\vdots$. However, in a later paper Challifour [5] did show the time-ordered Green's functions of $\vdots\exp(i\pi\lambda\sigma)\vdots$ exist for $|\lambda|$ small.

Interest in the model revived in the seventies in connection with work on the massive Thirring and sine-Gordon theories. It was claimed by Tapper [6] non-zero reflection occurs in second-order renormalized perturbation theory, contradicting the absence of reflection to all orders claimed in [1, 3]. This was subsequently refuted by several authors [7–10], who pointed out the inclusion of appropriate counterterms (needed also to respect Ward identities) does lead to vanishing reflection in second order. In the process a discrepancy between the S-operators of [1, 3] was resolved by Schroer, Truong, and Weisz, who also

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