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On the General Structure of Nonlinear Evolution Equations Integrable by the Two-Dimensional Matrix Spectral Problem

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Abstract. A generalization of the AKNS-technique to the two-dimensional arbitrary order matrix spectral problem is given. The general form of the integrable equations and their Bäcklund transformations in $1+2$ dimensions are found. The reduction problem is discussed.

I. Introduction

One of the main problems of the inverse scattering transform (IST) method is a problem of enumeration of the equations integrable by this method (see, e.g. [1, 2]). The simple and convenient description of a class of partial differential equations integrable by the one-dimensional second-order bundle

$$\frac{\partial \Psi}{\partial x} + \lambda A \Psi = P(x,t)\Psi \tag{1.0}$$

has been given by Ablowitz, Kaup, Newell, and Segur (AKNS) [3]. Then this approach (AKNS-approach) has been generalized to the problem (1.0) of arbitrary order [4–9] and to some other one-dimensional spectral problems [9–11]. The infinite-dimensional group of Bäcklund transformations for these classes of integrable equations has also been found [12, 7–9]. But up to now all the results obtained in the framework of the AKNS-approach [3–12] are concerned with equations in one spatial dimension.

The generalization of the AKNS-method to the case of several spatial dimensions is of indubitable interest. The applicability of the IST-method to the multidimensional equations has been demonstrated in [13, 14]. Various concrete two-dimensional and multidimensional evolution equations have been considered [13–18].

Multidimensional spectral problems possess a number of specific features. Nevertheless, as we shall see, the technique described in [7–9] permits a generalization to the $1+2$ dimensions (one time and two spatial dimensions) case.

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