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The Symmetry and Efimov's Effect in Systems of Three-Quantum Particles

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Abstract. The finiteness of the discrete spectrum of three body Schrödinger operators restricted to certain symmetry subspaces is proved. The symmetry subspaces are those associated with nonzero angular momentum and those associated with two or three identical fermions.

I. Introduction

Investigation of discrete spectrum energy operators of three-particle quantum systems without bound subsystems showed that the discrete spectrum of such systems can be infinite even if the potentials between the particles decrease arbitrarily rapidly (Elfimov's effect). This possibility is realized (under some additional conditions) for three-particle systems when energy operators h_α of two or three two-particle subsystems have virtual levels [1, 2] (see also [3]).

The presence of the operator h_α virtual level in this situation is connected with the existence of such solution φ of equation $h_\alpha \varphi = 0$ so that $|\nabla \varphi(x)| \in \mathscr{L}_2(R^3)$, $\varphi(x) \notin \mathscr{L}_2(R^3)$.

This article presents an investigation of the discrete spectrum of three-particle operators in some symmetry spaces. It is proved that in these spaces Efimov's effect is absent[1] so that for short-range potentials the discrete spectrum is finite.

Our proof of the finiteness is founded, mainly, on the investigation of virtual levels of two-particle Hamiltonians in the function subspaces from $\mathscr{L}_2(R^3) \ominus P^{(0)} \mathscr{L}_2(R^3)$ (see Sect. 3). It is established, in particular, (Theorem 3.1), that in these subspaces the presence of virtual levels of a two particle operator is connected with the presence of its zero eigenvalue as distinct from the case when the symmetry is not taken into account.

The main results of the work are Theorems 2.1–2.5 which are formulated in Sect. 2 and proved in Sect. 4. Theorem 3.1 is proved in Sect. 3. All auxiliary assertions and their proofs are in Sect. 5.

1 In physics articles this assertion appeared earlier [4] but this fact was not proved mathematically rigorously

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